These effects clearly are visible in Fig. 9.29(b). Not only does the image appear brighter than the original, but the sizes of dark features, such as the nostrils and the dark components of the studded rein extending from the ears down to the neck, have been reduced. Figure 9.29(c) shows the result of eroding the original image. Note the opposite effect to dilation. The eroded image is darker, and the sizes of small, bright features (such as the studs on the rein) were reduced. ■

### 9.6.3 Opening and Closing

The expressions for opening and closing of gray-scale images have the same form as their binary counterparts. The opening of image $f$ by subimage (structuring element) $b$, denoted $f \circ b$, is

$$f \circ b = (f \ominus b) \oplus b. \tag{9.6-4}$$

As in the binary case, opening is simply the erosion of $f$ by $b$, followed by a dilation of the result by $b$. Similarly, the closing of $f$ by $b$, denoted $f \bullet b$, is

$$f \bullet b = (f \oplus b) \ominus b. \tag{9.6-5}$$

The opening and closing for gray-scale images are duals with respect to complementation and reflection. That is,

$$(f \bullet b)^c = f^c \circ \hat{b}. \tag{9.6-6}$$

Because $f^c = -f(x, y)$, Eq. (9.6-6) can be written also as $-(f \bullet b) = (-f \circ \hat{b})$.

Opening and closing of images have a simple geometric interpretation. Suppose that we view an image function $f(x, y)$ in 3-D perspective (like a relief map), with the $x$- and $y$-axes being the usual spatial coordinates and the third axis being gray-level values. In this representation, the image appears as a discrete surface whose value at any point $(x, y)$ is the value of $f$ at those coordinates. Suppose that we open $f$ by a spherical structuring element, $b$, viewing this element as a "rolling ball." Then the mechanics of opening $f$ by $b$ may be interpreted geometrically as the process of pushing the ball against the *underside* of the surface, while at the same time rolling it so that the entire underside of the surface is traversed. The opening, $f \circ b$, then is the surface of the highest points reached by any part of the sphere as it slides over the *entire* undersurface of $f$.

Figure 9.30 illustrates this concept. Figure 9.30(a) shows a scan line of a gray-scale image as a continuous function to simplify the illustration. Figure 9.30(b) shows the rolling ball in various positions, and Fig. 9.30(c) shows the complete result of opening $f$ by $b$ along the scan line. All the peaks that were narrow with respect to the diameter of the ball were reduced in amplitude and sharpness. In practical applications, opening operations usually are applied to remove small (with respect to the size of the structuring element) light details, while leaving the overall gray levels and larger bright features relatively undisturbed. The initial erosion removes the small details, but it also darkens the image. The subsequent dilation again increases the overall intensity of the image without reintroducing the details totally removed by erosion.



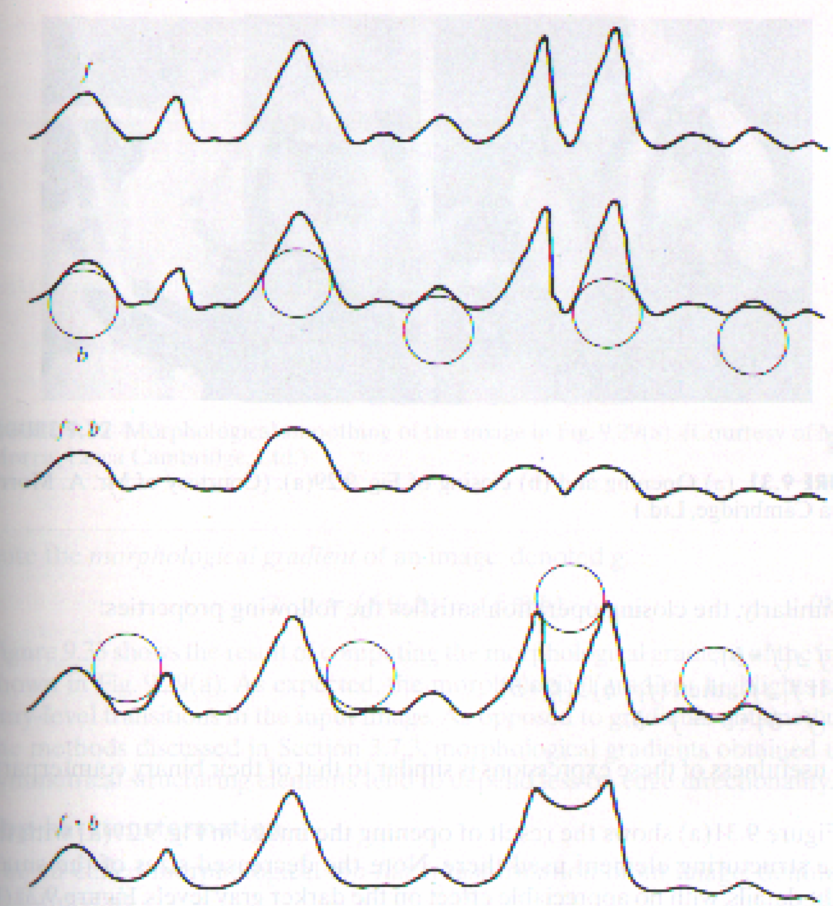


**FIGURE 9.30** (a) A gray-scale scan line. (b) Positions of rolling ball for opening. (c) Result of opening. (d) Positions of rolling ball for closing. (e) Result of closing.

Figures 9.30(d) and (e) show the result of closing $f$ by $b$. Here, the ball slides on top of the surface, and peaks essentially are left in their original form (assuming that their separation at the narrowest point exceeds the diameter of the ball). In practice, closing is generally used to remove dark details from an image, while leaving bright features relatively undisturbed. The initial dilation removes the dark details and brightens the image, and the subsequent erosion darkens the image without reintroducing the details removed by dilation. It is of interest to compare Fig. 9.30 with Figs. 9.8 and 9.9.

The gray-scale opening operation satisfies the following properties:

**(i)** $(f \circ b) \dashv f$.
**(ii)** If $f_1 \dashv f_2$, then $(f_1 \circ b) \dashv (f_2 \circ b)$.
**(iii)** $(f \circ b) \circ b = f \circ b$.

The notation $e \dashv r$ is used to indicate that the domain of $e$ is a subset of the domain of $r$, and also that $e(x, y) \leq r(x, y)$ for any $(x, y)$ in the domain of $e$.