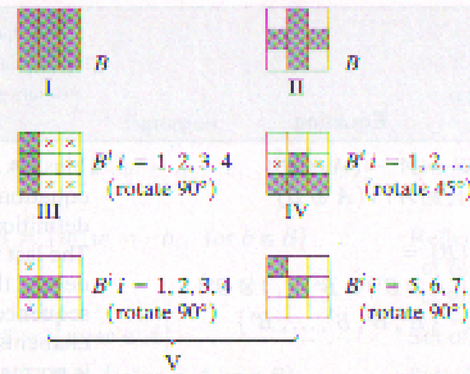


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

9.6 Extensions to Gray-Scale Images

In this section we extend to gray-scale images the basic operations of dilation, erosion, opening, and closing. We then use these operations to develop several basic gray-scale morphological algorithms. In particular, we develop algorithms for boundary extraction via a morphological gradient operation, and for region partitioning based on texture content. We also discuss algorithms for smoothing and sharpening, which often are useful as pre- or postprocessing steps.

Throughout the discussions that follow, we deal with digital image functions of the form $f(x, y)$ and $b(x, y)$, where $f(x, y)$ is the input image and $b(x, y)$ is a structuring element, itself a subimage function. The assumption is that these functions are discrete in the sense introduced in Section 2.4.2. That is, if Z denotes the set of real integers, the assumption is that (x, y) are integers from $Z \times Z$ and that f and b are functions that assign a gray-level value (a real number from the set of real numbers, R) to each distinct pair of coordinates (x, y). If the gray levels also are integers, Z replaces R.

9.6.1 Dilation

Gray-scale dilation of f by b, denoted $f \oplus b$, is defined as

$$(f \oplus b)(s, t) = \max\{f(s - x, t - y) + b(x, y) \mid (s - x), (t - y) \in D_f; (x, y) \in D_b\} \quad (9.6\text{-}1)$$

where D_f and D_b are the domains of f and b, respectively. Keep in mind that f and b are functions, rather than sets, as is the case in binary morphology.

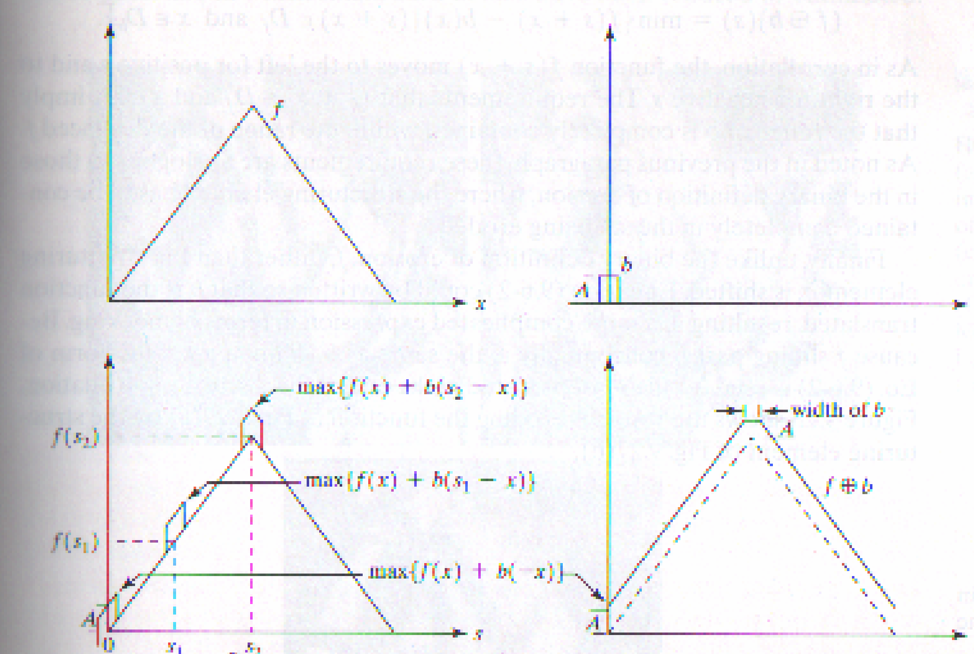
The condition that $(s - x)$ and $(t - y)$ have to be in the domain of f, and x and y have to be in the domain of b, is analogous to the condition in the binary definition of dilation, where the two sets have to overlap by at least one element. Note also that the form of Eq. (9.6-1) is similar to 2-D convolution [Eq. (4.2-30)], with the max operation replacing the sums of convolution and the addition replacing the products of convolution.

We illustrate the notation and mechanics of Eq. (9.6-1) by means of simple 1-D functions. For functions of one variable, Eq. (9.6-1) reduces to the expression

$$(f \oplus b)(s) = \max\{f(s - x) + b(x) \mid (s - x) \in D_f \text{ and } x \in D_b\}.$$

Recall from the discussion of convolution that $f(-x)$ is simply $f(x)$ mirrored with respect to the origin of the x axis. As in convolution, the function $f(s - x)$ moves to the right for positive s, and to the left for negative s. The requirements that the value of $(s - x)$ has to be in the domain of f and that the value of x has to be in the domain of b imply that f and b overlap. As noted in the previous paragraph, these conditions are analogous to the requirement in the binary definition of dilation, where the two sets have to overlap by at least one element. Finally, unlike the binary case, f, rather than the structuring element b, is shifted. Equation (9.6-1) could be written so that b undergoes translation instead of f. However, if D_b is smaller than D_f (a condition almost always found in practice), the form given in Eq. (9.6-1) is simpler in terms of indexing and achieves the same result. Conceptually, f sliding by b is really no different than b sliding by f. In fact, although this equation is easier to implement, the actual mechanics of gray-scale dilation are easier to visualize if b is the function that slides past f.

An example is shown in Fig. 9.27. Note that at each position of the structuring element the value of dilation at that point is the maximum of the sum of f



a b
c d

FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).