TABLE 11.2 Texture measures for the subimages shown in Fig. 11.22.

Texture	Mean	Standard deviation	R (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

This gives a rough idea of whether the gray levels are biased toward the dark or light side of the mean. In terms of texture, the information derived from the third moment is useful only when variations between measurements are large. Looking at the measure of uniformity, we again conclude that the first subimage is smoother (more uniform than the rest) and that the most random (lowest uniformity) corresponds to the coarse texture. This is not surprising. Finally, the entropy values are in the opposite order and thus lead us to the same conclusions as the uniformity measure did. The first subimage has the lowest variation in gray level and the coarse image the most. The regular texture is in between the two extremes with respect to both these measures. ■

Measures of texture computed using only histograms suffer from the limitation that they carry no information regarding the relative position of pixels with respect to each other. One way to bring this type of information into the texture-analysis process is to consider not only the distribution of intensities, but also the positions of pixels with equal or nearly equal intensity values.

Let P be a position operator and let $\mathbf{A}$ be a $k \times k$ matrix whose element a_{ij} is the number of times that points with gray level z_i occur (in the position specified by P) relative to points with gray level z_j, with $1 \leq i, j \leq k$. For instance, consider an image with three gray levels, $z_1 = 0$, $z_2 = 1$, and $z_3 = 2$, as follows:

$$\begin{matrix} 0 & 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{matrix}$$

Defining the position operator P as "one pixel to the right and one pixel below" yields the following 3×3 matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$

where, for example, a_{11} (top left) is the number of times that a point with level $z_1 = 0$ appears one pixel location below and to the right of a pixel with the same gray level, and a_{13} (top right) is the number of times that a point with level $z_1 = 0$ appears one pixel location below and to the right of a point with gray level $z_3 = 2$. The size of $\mathbf{A}$ is determined by the number of distinct gray levels in the input image. Thus application of the concepts discussed in this section usually requires that intensities be requantized into a few gray-level bands in order to keep the size of $\mathbf{A}$ manageable.

Let n be the total number of point pairs in the image that satisfy P (in the preceding example $n = 16$, the sum of all values in matrix $\mathbf{A}$). If a matrix $\mathbf{C}$ is formed by dividing every element of $\mathbf{A}$ by n, then c_{ij} is an estimate of the joint probability that a pair of points satisfying P will have values (z_i, z_j). The matrix $\mathbf{C}$ is called the *gray-level co-occurrence matrix*. Because $\mathbf{C}$ depends on P, the presence of given texture patterns may be detected by choosing an appropriate position operator. For instance, the operator used in the preceding example is sensitive to bands of constant intensity running at −45°. (Note that the highest value in $\mathbf{A}$ was $a_{11} = 4$, partially due to a streak of points with intensity 0 and running at −45°.) More generally, the problem is to analyze a given $\mathbf{C}$ matrix in order to categorize the texture of the region over which $\mathbf{C}$ was computed. A set of descriptors useful for this purpose includes the following:

1. Maximum probability

$$\max_{i,j}(c_{ij})$$

2. Element difference moment of order k

$$\sum_i \sum_j (i - j)^k c_{ij}$$

3. Inverse element difference moment of order k

$$\sum_i \sum_j c_{ij}/(i - j)^k \qquad i \neq j$$

4. Uniformity

$$\sum_i \sum_j c_{ij}^2$$

5. Entropy

$$-\sum_i \sum_j c_{ij} \log_2 c_{ij}$$

The basic idea is to characterize the "content" of $\mathbf{C}$ via these descriptors. For example, the first property gives an indication of the strongest response to P. The second descriptor has a relatively low value when the high values of $\mathbf{C}$ are near the main diagonal, because the differences $(i - j)$ are smaller there. The third descriptor has the opposite effect. The fourth descriptor is highest when the c_{ij}s are all equal. As noted previously, the fifth descriptor is a measure of randomness, achieving its highest value when all elements of $\mathbf{C}$ are maximally random.

One approach for using these descriptors is to "teach" a system representative descriptor values for a set of different textures. The texture of an unknown region is then subsequently determined by how closely its descriptors match those stored in the system memory. We discuss matching in more detail in Chapter 12.