and b in the interval spanned by b. The general effect of performing dilation on a gray-scale image is twofold: (1) If all the values of the structuring element are positive, the output image tends to be brighter than the input. (2) Dark details either are reduced or eliminated, depending on how their values and shapes relate to the structuring element used for dilation.

9.6.2 Erosion

Gray-scale erosion, denoted $f \ominus b$, is defined as

$$(f \ominus b)(s,t) = \min\{f(s+x, t+y) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\} \quad (9.6\text{-}2)$$

where D_f and D_b are the domains of f and b, respectively. The condition that $(s+x)$ and $(t+y)$ have to be in the domain of f, and x and y have to be in the domain of b, is analogous to the condition in the binary definition of erosion, where the structuring element has to be completely contained by the set being eroded. Note that the form of Eq. (9.6-2) is similar in form to 2-D correlation [Eq. (4.6-30)], with the min operation replacing the sums of correlation and subtraction replacing the products of correlation.

We illustrate the mechanics of Eq. (9.6-2) by eroding a simple 1-D function. For functions of one variable, the expression for erosion reduces to

$$(f \ominus b)(s) = \min\{f(s+x) - b(x) \mid (s+x) \in D_f \text{ and } x \in D_b\}.$$

As in correlation, the function $f(s+x)$ moves to the left for positive s and to the right for negative s. The requirements that $(s+x) \in D_f$ and $x \in D_b$ imply that the range of b is completely contained within the range of the displaced f. As noted in the previous paragraph, these requirements are analogous to those in the binary definition of erosion, where the structuring element has to be contained completely in the set being eroded.

Finally, unlike the binary definition of erosion, f, rather than the structuring element b, is shifted. Equation (9.6-2) could be written so that b is the function translated, resulting in a more complicated expression in terms of indexing. Because f sliding past b conceptually is the same as b sliding past f, the form of Eq. (9.6-2) is used for the reasons stated at the end of the discussion on dilation. Figure 9.28 shows the result of eroding the function of Fig. 9.27(a) by the structuring element of Fig. 9.27(b).

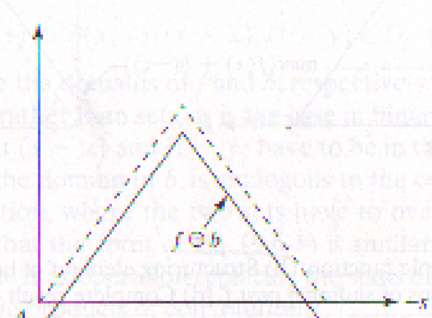


FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).

Equation (9.6-2) indicates that erosion is based on choosing the minimum value of $(f - b)$ in the interval defined by the shape of the structuring element. The general effect of performing erosion on a gray-scale image is twofold: (1) If all the elements of the structuring element are positive, the output image tends to be darker than the input image. (2) The effect of bright details in the input image that are smaller in area than the structuring element is reduced, with the degree of reduction being determined by the gray-level values surrounding the bright detail and by the shape and amplitude values of the structuring element itself.

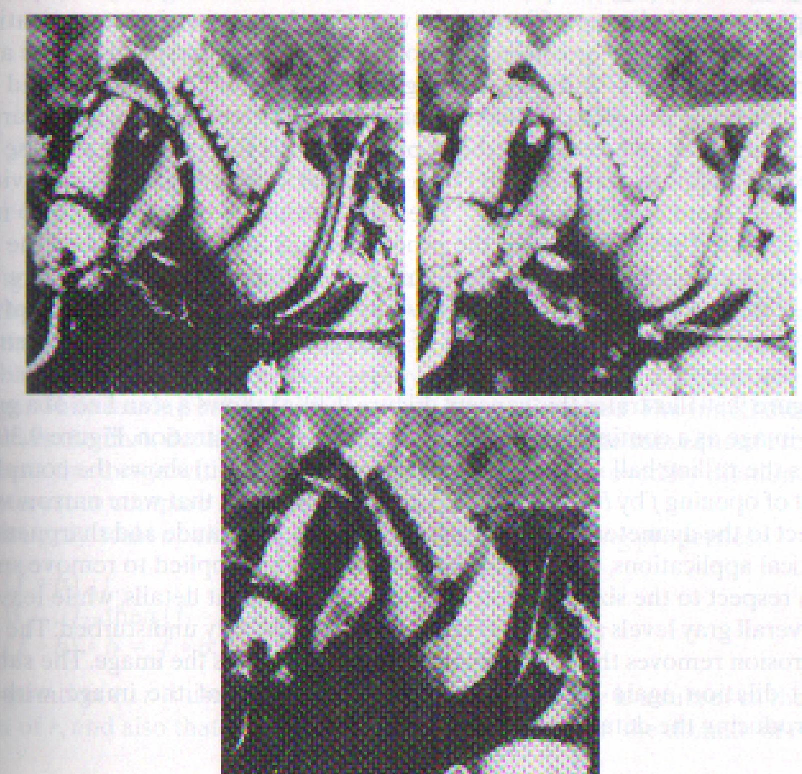
Gray-scale dilation and erosion are duals with respect to function complementation and reflection. That is,

$$(f \ominus b)^c(s,t) = (f^c \oplus \hat{b})(s,t) \quad (9.6\text{-}3)$$

where $f^c = -f(x,y)$ and $\hat{b} = b(-x,-y)$. Except as needed for clarity, we simplify the notation in the following discussions by omitting the arguments of all functions.

EXAMPLE 9.9: Illustration of dilation and erosion on a gray-scale image.

■ Figure 9.29(a) shows a simple 512×512 gray-scale image, and Fig. 9.29(b) shows the result of dilating this image with a "flat-top" structuring element in the shape of a parallelepiped of unit height and size 5×5 pixels. Based on the preceding discussion, dilation is expected to produce an image that is brighter than the original and in which small, dark details have been reduced or eliminated.



a b
c

FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)