a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Similarly, the closing operation satisfies the following properties:

(i) $f \subset (f \bullet b)$.
(ii) If $f_1 \subset f_2$, then $(f_1 \bullet b) \subset (f_2 \bullet b)$.
(iii) $(f \bullet b) \bullet b = f \bullet b$.

The usefulness of these expressions is similar to that of their binary counterparts.

EXAMPLE 9.10: Illustration of gray-scale opening and closing.

■ Figure 9.31(a) shows the result of opening the image in Fig. 9.29(a) with the same structuring element used there. Note the decreased sizes of the small, bright details, with no appreciable effect on the darker gray levels. Figure 9.31(b) shows the closing of Fig. 9.29(a). Note the decreased sizes of the small, dark details, with relatively little effect on the bright features. ■

9.6.4 Some Applications of Gray-Scale Morphology

We conclude the discussion of morphological techniques by presenting in some detail various applications of gray-scale morphology. Unless stated otherwise, all the images shown are of size 512×512 and were processed by using the structuring element discussed in connection with Fig. 9.29.

Morphological smoothing

One way to achieve smoothing is to perform a morphological opening followed by a closing. The net result of these two operations is to remove or attenuate both bright and dark artifacts or noise. Figure 9.32 shows a smoothed version of the image shown in Fig. 9.29(a).

Morphological gradient

In addition to the operations discussed earlier in connection with the removal of small dark and bright artifacts, dilation and erosion often are used to compute the *morphological gradient* of an image, denoted g:

$$g = (f \oplus b) - (f \ominus b). \tag{9.6-7}$$

Figure 9.33 shows the result of computing the morphological gradient of the image shown in Fig. 9.29(a). As expected, the morphological gradient highlights sharp gray-level transitions in the input image. As opposed to gradients obtained using the methods discussed in Section 3.7.3, morphological gradients obtained using symmetrical structuring elements tend to depend less on edge directionality.

FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Top-hat transformation

The so-called morphological *top-hat* transformation of an image, denoted h, is defined as

$$h = f - (f \circ b) \tag{9.6-8}$$

FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)